**STAT 351 – Homework 4  - Due by 11.59 pm on Thursday, April 29**

**You are encouraged to work on this assignment in groups. However, you cannot write exactly same explanations.**

**If I see the exact same explanation on two or more assignments, then I’ll give zero points for those assignments.**

**Show your work before the final answer to get full points.**

**Problem 1:**

Heat must be removed from a system according to how fast it is generated. Suppose the system has 10 components each of which is active with probability 0.15, independently of the others. The design of the heat removal system requires finding the following information.

**(keep 4 or more decimal places in your answers)**:

(a) Probability that none of the components in this system is active.

(b) Probability that at least one component in this system is active.

(c) Probability that exactly 3 components in this system are active.

(d) Probability that more than 2 components in this system are active.

(e) Expected number of active components in this system.

**Problem 2:**

The probability that a semiconductor wafer contains a large particle is 0.025 and the wafers contain a large particle independently. Compute the following probabilities: (**keep 4 or more decimal places**):

(a) Probability that 5 wafers need to be analyzed in order to detect the 1st wafer with a large particle of contamination.

(b) Probability that less than 5 wafers need to be analyzed in order to detect the 1st wafer with a large particle of contamination.

(c) Probability that at least 5 wafers need to be analyzed in order to detect the 1st wafer with a large particle of contamination.

**Problem 3:**

Assume that the number of messages input to a communication channel in an interval of duration 2 seconds has a Poisson distribution with mean 0.6.

Compute the probabilities of the following events (**keep 4 or more decimal places**):

(a) Exactly four messages will arrive during a 10 s interval.

(b) At most 10 messages arrive in a 20s interval.

(c) The number of message arrivals in an interval of 5 s duration is from 4 to 6.

**Problem 4:** A test for the presence of a certain disease has probability 0.05 of giving a false-positive result and probability 0.04 of giving a false-negative result.

(a) Suppose that randomly selected individuals who do not have the disease are tested until one individual gets positive test results. Compute the average (or expected) number of such individuals needs to be tested in this study.

(b) Suppose that randomly selected 15 individuals who have the disease are tested. Compute the probability that 2 or less individuals receive negative results.

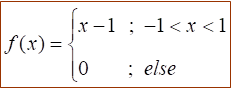
(c) Suppose that randomly selected individuals who do not have the disease are tested until three individuals get positive test results. Let W be the number of such individuals needs to be tested in this study.

(i) Select one of the following answers for the probability distribution of W:

**Binomial Geometric Poisson None of these**

(ii) Provide your reasoning.

**Problem 5:** Explain whether the following function is a probability density function (pdf) or not:



**Problem 6:**

The pdf of a random variable is given below:



Find the value of k so that f(x) is a pdf.

**Problem 7:**

The proportion of people in US who dine out on Valentine’s Day is denoted by X and it has the following pdf:

Find the probability that less than 40% of the people will dine out next year. Simplify your answer and **keep at least 4 decimal places**.